## Supplemental Activity: Vectors and Forces

Objective: To use a force table to test equilibrium conditions.

Required Equipment: Force Table, Pasco Mass and Hanger Set, String, Ruler, Polar Graph Paper, Protractor, Colored Pencils

## Theory:

When you push on something, you know that you can vary the size, or magnitude, of the exerted force by varying your muscular exertion. You can also vary the direction that you push, exerting a force in practically any direction. For example, if you and another person push on a stalled car to start it moving, you know the direction you both push is as important as the magnitude of your push. If you are both side by side, pushing in the same direction, your forces add to some net force in the same direction. If you push in parallel but opposite directions, however, one force will cancel the other and the net force will be zero. In between the two extremes of parallel forces in the same direction and parallel forces in opposite directions, two people pushing at various directions will produce net forces somewhere between a net force of zero and a maximum net force from adding the two forces together. It should be clear that a force has both a magnitude, or size, and it has a direction. Quantities that have both magnitude and direction like this are called vectors. Velocity, acceleration, and a force all have magnitude and direction, so they are vector quantities. Quantities that have magnitude only are called scalars. Volume, temperature, and area all have magnitude only, so they are scalar quantities. Scalar quantities can be added by ordinary arithmetic, which is called scalar addition. Scalar addition will not work with vectors, however, since these quantities have a property of direction.

The vector sum of two or more vectors is called the resultant. There are two basic methods used to find a resultant: (1) the graphical method and (2) the analytical component method. In the graphical method a vector is illustrated by drawing a straight line with an arrowhead pointing in the direction of the vector. The length of the arrow is drawn to a scale to represent the magnitude of the vector quantity. For example, when two displacement vectors are drawn on the same diagram, the arrow representing the larger displacement must be longer than the arrow representing the shorter displacement. The figure below shows the resultant of 103 N when a force of 50 N is applied toward the North and a force of 90 N is applied to the East.


According to Newton's laws of motion, an object at rest with two or more forces acting on it must have a vector sum, or resultant, of zero. An object with two or more forces with a resultant of zero is said to be in equilibrium. In equilibrium there is a vector called the equilibrant that cancels the effects of the other vectors. The equilibrant always has the same magnitude but opposite direction as the resultant. In this experiment you will investigate resultants, equilibrants, and other force vectors in equilibrium, using the graphical method of vector addition.

Procedure: Examine the force table that you will use and note how the pulleys can be moved around the table to various positions. A string is tied to a ring at the center, runs over a pulley, and down to a mass hanger. You can create forces of different magnitudes and directions on this ring by varying the mass on the hangers and by moving the pulleys.


Trial 1: If 100 grams is placed at $0^{\circ}$ and 50 grams is placed at $90^{\circ}$ where must a third mass be placed to put the system in equilibrium? First find the answer using graphical means and then use the force table to verify your result.

Procedure:

1. Record the values for $m_{1}$ and $m_{2}$ in grams. Although force is usually measured in Newton's, we will use the unit of grams for force. This is acceptable becauseg is the same for all masses
2. Using polar graph paper, a protractor, and a ruler draw vector arrows for the force on each string on the force table, and label them $\mathbf{F}_{1}$ and $\mathbf{F}_{2}$. Chose a convenient scale that will permit you to draw the longest arrows possible. Record the scale somewhere off to the side of the graph paper.
3. Use the parallelogram rule to draw in the resultant vector. Measure the length of the resultant vector and multiply it by your scale to determine its magnitude. Record this measurement as the graphical resultant of $\mathbf{F}_{1}$ and $\mathbf{F}_{2}$. Measure the angle of the resultant vector using a protractor and record it in your data table.
4. Now draw the equilibrium vector. The equilibrium vector is the third vector needed to keep the system in equilibrium. The equilibrium vector will have the same magnitude as the resultant vector but it will be $180^{\circ}$ from the resultant. Record the magnitude and direction in the data table as the graphical equilibrium vector.
5. Now set up the force table with the given masses. Make sure the table is level, and adjust each string so it is level with the force table and aligned correctly on its pulley. Also check the center ring to make sure the string is pointing to the center of the ring, not the edge or elsewhere. Check this from time to time and move the string knot as necessary to ensure alignment.
6. Verify your results for the equilibrium vector by adding a third pulley to the force table and adjust its position until the pole is in the center of the metal ring. Record your results in the data table andcalculatea percent difference.
7. An example is given below. In this sample, $m_{1}$ was 50 grams and $m_{2}$ was 25 grams.

| Data Table: Adding Two Vector Forces (Trial 1) |  |  |
| :--- | :---: | :---: |
| String 1 | Force(9) | Angle (degrees) |
| String 2 | 50 | $0^{\circ}$ |
| Graphical Results: Resultant of F 1 and F | 25 | $90^{\circ}$ |
| Graphical Equilibrium Vector 2 | 56.5 | $27^{\circ}$ |
| Experimental Equilibrium Vector | 55.9 | $207^{\circ}$ |



Trial 1: $\mathrm{m}_{1}=100$ grams @ $0^{\circ}$ and $\mathrm{m}_{2}=50$ grams at $90^{\circ}$.
Data Table: Adding Two Vector Forces (Trial 1)

|  | Force(g) | Angle (degrees) |
| :--- | :--- | :--- |
| String $\mathbf{1}$ |  |  |
| String 2 |  |  |
| Graphical Results: Resultant of F $_{1}$ and $\mathbf{F}$ |  |  |
| Graphical Equilibrium Vector | 2 |  |
| Experimental Equilibrium Vector |  |  |



Compute the percent difference between your two equilibrium results above and write your result in the box to the right. Assume that the graphical result is the theoretical value.


Trial 2: Repeat steps $2-6$ with $m_{1}=35$ grams @ $0^{\circ}$ and $m_{2}=70$ grams at $30^{\circ}$.

| Data Table: Adding Two Vector Forces (Trial 2) |  |  |
| :--- | :---: | :---: |
| String 1 | Force(g) | Angle (degrees) |
| String 2 |  |  |
| Graphical Results: Resultant of F $_{1}$ and F |  |  |
| Graphical Equilibrium Vector |  |  |
| Experimental Equilibrium Vector |  |  |



Note: Choose a scale where the vectors will be between two and five centimeters long.
$1 \mathrm{~cm}=$ $\qquad$ grams
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Compute the percent difference between your two equilibrium results above and write your result in the box to the right. Assume that the graphical result is the theoretical value.


Trial 3: Repeat steps $2-6$ with $m_{1}=35$ grams @ $0^{\circ}$ and $m_{2}=70$ grams at $60^{\circ}$.
Data Table: Adding Two Vector Forces (Trial 3)

|  | Force(g) | Angle (degrees) |
| :--- | :--- | :--- |
| String 1 |  |  |
| String 2 |  |  |
| Graphical Results: Resultant of F $_{1}$ and F |  |  |
| Graphical Equilibrium Vector |  |  |
| Experimental Equilibrium Vector |  |  |



Compute the percent difference between your two equilibrium results above and write your result in the box to the right. Assume that the graphical result is the theoretical value.


Trial 4: Repeat steps $2-6$ with $m_{1}=35$ grams @ $0^{\circ}$ and $m_{2}=70$ grams at $90^{\circ}$.

| Data Table: Adding Two Vector Forces (Trial 4) |  |  |
| :--- | :---: | :---: |
| String 1 | Force(g) | Angle (degrees) |
| String 2 |  |  |
| Graphical Results: Resultant of F $_{1}$ and F |  |  |
| Graphical Equilibrium Vector |  |  |
| Experimental Equilibrium Vector |  |  |



Compute the percent difference between your two equilibrium results above and write your result in the box to the right. Assume that the graphical result is the theoretical value.


Trial 5: Repeat steps $2-6$ with $m_{1}=35$ grams @ $0^{\circ}$ and $m_{2}=70$ grams at $150^{\circ}$.
Data Table: Adding Two Vector Forces (Trial 5)

|  | Force(g) | Angle (degrees) |
| :--- | :--- | :--- |
| String 1 |  |  |
| String 2 |  |  |
| Graphical Results: Resultant of F $_{1}$ and F |  |  |
| Graphical Equilibrium Vector |  |  |
| Experimental Equilibrium Vector |  |  |

90


Note: Choose a scale where the vectors will be between two and five centimeters long.
$1 \mathrm{~cm}=$ $\qquad$ grams

Compute the percent difference between your two equilibrium results above and write your result in the box to the right. Assume that the graphical result is the theoretical value.

## Additional Questions

1. In general, when the angle between two vectors increases, what happens to the magnitude of the resultant? Does the resultant increase or decrease?
2. If you want to add two vectors and get the largest possible resultant what angle should the angle between the vectors be?
3. If a 30 unit vector is added to a 70 unit vector what is the magnitude of the largest possible resultant?
4. If you want to add two vectors and get the smallest possible resultant what angle should the angle between the vectors be?
5. If a 30 unit vector is added to a 70 unit vector what would the magnitude of the smallest possible resultant?
6. In each of the cases below, draw and label the vector needed to keep the system in equilibrium.


Figure 1


