

Experiment 11

Moment of Inertia

A rigid body composed of concentric disks is constrained to rotate about its axis of symmetry. The moment of inertia is found by two methods and the results are compared. In the first method, the moment of inertia is determined *theoretically* by applying the formula for the moment of inertia of a single disk to each of four disks and adding the results. In the second method, the moment of inertia is determined *experimentally* by measuring the acceleration produced by a constant torque on the body.

The experimental determination of the moment of inertia is only valid if friction is negligible. In Part III of the experiment, an estimate of the angular acceleration due to friction is obtained. The validity of the above approximation is the examined.

Theory, Part I

Consider a pulley made of four disks.

The moment of inertia of a homogeneous disk about the axis of symmetry is

$$I_{\text{disk}} = \frac{1}{2} M R^2,$$

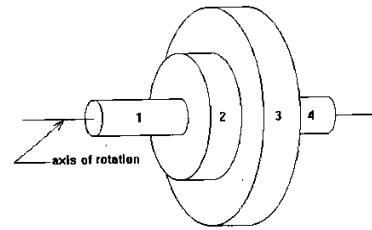


Figure 1. The body consists of four concentric disks.

where M is the mass of the disk and R is the radius. The moment of inertia of a rigid system of concentric disks is then

$$I = \frac{1}{2} M_1 R_1^2 + \frac{1}{2} M_2 R_2^2 + \dots = \sum \frac{1}{2} M_i R_i^2, \quad (1)$$

where the sum extends over all disks, each of which has mass M_i and radius R_i . If the mass density, ρ , is uniform (i.e., constant throughout the body), the mass of each disk is given by

$$M_i = \rho V_i = \rho w_i \pi R_i^2,$$

where V_i is the volume of each disk and w_i is the width. Substituting this into (1), the moment of inertia of each disk is then

$$I_i = \frac{\pi \rho}{2} w_i R_i^4, \quad (2)$$

and the total theoretical moment of inertia of the rotating body is

$$I_{the} = \frac{\pi\rho}{2} \sum w_i R_i^4. \quad (3)$$

If the density together with the width and the radius of each disk are known, then the moment of inertia of the body can be computed. The moment of inertia determined in this manner will be referred to as the theoretical value (I_{theo}).

Theory, Part II

The experimental value of the moment of inertia can also be found by exerting a constant torque on the body. A mass, m , is attached to a string which is wrapped around the body at some radius, R . (Refer to Figure 2.) R will be one of the disk radii, R_i . If m is released from rest and falls a distance, d , during a time, t , the acceleration of m is given by

$$a = \frac{2d}{t^2}. \quad (4)$$

Once the value of the acceleration is known, the moment of inertia is determined by

$$I_{exp} = m R^2 \left(\frac{g}{a} - 1 \right), \quad (5) \text{ assuming that friction in the supports is negligible.}$$

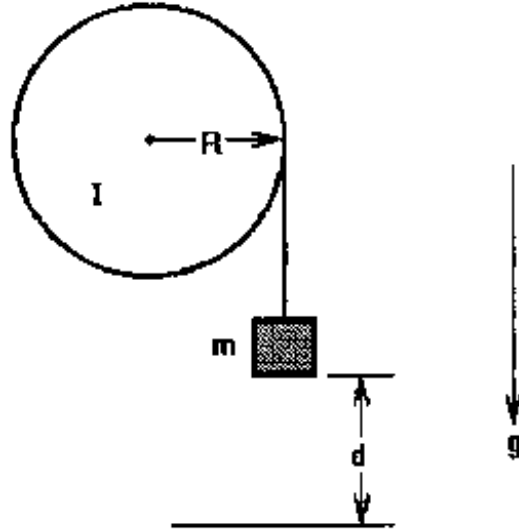


Figure 2. Experimental determination of moment of inertia.

Theory, Part III

A rough measure of friction in the supports can easily be found. Suppose the body (without the mass) is initially spun and N revolutions occur during the time T_N required for the body to come to rest. Assuming the angular acceleration due to the resistance is constant, its magnitude is then given by

$$\alpha_f = \frac{4\pi N}{T_N^2}. \quad (6)$$

This should be approximately true for the apparatus. The expression (6) represents, in reality, the average value of the angular acceleration due to friction.

The magnitude of the average resistive torque due to friction in the supports is $\tau_f = I\alpha_f$. Including this torque in the derivation of the experimental moment of inertia (5), the corrected result is

$$I_{\text{exp}} = m R^2 \left(\frac{g}{a} - 1 \right) \frac{I}{\left(I + \frac{\alpha_f}{a/R} \right)}, \quad (7)$$

This is assuming the resistive torque is constant. In the experimental determination of the moment of inertia, when m is attached to the body, the resistive torque will have an additional contribution that is proportional to the tension of the string, which connects m to the body. This contribution is negligible, however, if $m \ll M_{\text{total}}$. According to (7) the validity of (5) rests upon the degree to which the following is true:

$$\frac{\alpha_f}{a/R} \ll 1. \quad (8)$$

Apparatus

- o mounted rotational body
- o vernier caliper
- o string
- o masking tape
- o 100-gram slotted weight
- o two meter stick
- o stopwatch

The pulley is made of aluminum and consists of four concentric disks. (Refer to Figure 2.) The density of the material is given by the total mass divided by the total volume. The result is

$$\rho = 2.70 \times 10^3 \text{ kg} \cdot \text{m}^{-3}. \quad (10)$$

The string attached to the mass m supplies a force that can be applied at any of the different radii of the pulley. The string can be attached to the cylinder with some masking tape.

Procedure, Parts I & II

- 1) Using the vernier caliper, measure and record the width, w , and diameter, d , of each disk. Obtain 3 or 4 significant figures. Write down the uncertainties for each measurement. Number the disks according to Figure 2.
- 2) Cut a piece of string long enough to allow the weight hanger to reach the floor. Attach a 100-gram weight to the string. This is mass m .
- 3) Attach the other end of the string to the rim of one of the disks. Ensure that the string will not slip. Rotate the body, allowing the string to wrap around the disk without overlapping. Using the two-meter stick, position the bottom of the weight so that its vertical distance d from the floor is approximately 1 meter
- 4) Steady the weight. Release the body and measure the time, t , required for the weight hanger to strike the floor. The timing must be performed very *carefully*; start the timer

exactly when the body is released and stop it just when the body strikes the floor. Do not permit the string to become tangled in the supports. Perform two more trials using different students as timers and obtain an average and uncertainty for this measurement.

- 5) Repeat the procedure on another radius. Your data will consist of three “time-to-fall” measurements for each different radius. Note that the values of m and d are to remain constant.

Procedure, Part III (optional)

- 1) Remove the string from the body, and place a small piece of tape on the rim of the largest disk. The tape will serve as a reference mark to be used in counting revolutions.
- 2) Spin the body as fast as possible while still being able to count the revolutions. The angular velocity will then be comparable to a typical final angular velocity in Part I.
- 3) Start the time and count the number, N , of revolutions during the time, T_N , required for the body to come to rest. Record the values of N and T_N . The data need only be approximate and only one trial will suffice.

Analysis, Part I

Construct a data table that contains the width, diameter, radius, and moment of inertia of each disk, using (2) and (10). Use SI (MKS) units. Include the value of ρ in the title of the table. Number the disks according to Figure 2. Determine and clearly display the value of the total theoretical moment of inertia, I_{theo} , to three significant figures.

Analysis, Part II

For each radius, R , at which a force was applied, compute the average value of the time-of-fall, t . Using (4), compute the acceleration for each value of R . Then, using (5), compute I for each set of values of R and a . Use SI units. Construct a data table that contains the values of R , t , A , and I_{exp} . (This table should be separate from the table for the theoretical moment of inertia.) Include both the measured and averaged times-of-fall. Since the values of m and d are constant, they should be included in the title of the table. Determine and display the values obtained for I_{exp} .

Also determine the uncertainties for the theoretical value and for the experimental values. For the experimental results use the fractional uncertainty formula.

$$\delta I = I \left[\frac{2\delta \bar{t}}{\bar{t}} + \frac{2\delta R}{R} + \frac{\delta d}{d} \right].$$

We assume here that the uncertainty in the mass m is negligible.

Results

Report in a table of results the experimental and theoretical values of the moment of inertia, their uncertainties and the percentage difference between the theoretical and each experimental value. Find the percentage difference using the equation:

$$\frac{|I_{\text{exp}} - I_{\text{the}}|}{I_{\text{the}}} \times 100\%.$$

Include a one-dimensional error bar comparing the single theoretical value with the experimentally determined values.

Analysis, Part III (optional)

Using (6), compute and display the approximate value of the angular acceleration, α_f , due to friction. Express the result in units of rad/sec^2 . For each value of R used, compute the angular acceleration, a/R , and the corrected I_{exp} values. Include these values in the results table and the error bar graph in Part I.

Questions and Conclusion

- 1) In the theory section of your lab report, draw free body diagrams for m and for the pulley, and derive (4) and (5). Explain your steps.
- 2) In the theory section of your lab report, derive (6) and (7). Explain your steps.
- 3) In the experimental method, explain why the acceleration is greater when you attach the string to a larger radius.
- 4) Your two experimental results should each give the same value for the entire “ I ” of the object, and not just the “ I_i ” of the disk to which the string was attached. Explain.
- 5) Was the experiment a success? (Did the theoretical and experimental values agree to the degree of uncertainty). Were there systematic errors that could explain the differences?
- 6) Did the two different experimental results (derived from wrapping the string around different radii) agree with each other? Can you think of any systematic errors that could explain the differences?
- 7) Does the friction correction improve the agreement between theoretical and experimental values? Explain.